

## AN EXPLORATION OF EARLY CHILDHOOD EDUCATORS' PEDAGOGICAL CONTENT KNOWLEDGE OF MATHEMATICS: A MIXED METHODS STUDY

**Assoc. Prof. Dr. Ali Yiğit KUTLUCA**

İstanbul Aydin University, İstanbul, Türkiye

ORCID: 0000-0002-1341-3432

**Mikail DURSUN**

Ministry of National Education, İstanbul, Türkiye

ORCID: 0009-0006-0834-0669

### **Abstract**

The primary objective of this study was to explore how early childhood educators' pedagogical content knowledge (Math-PCK) related to mathematics influences their conceptualization of activity plans. The secondary objective was to determine the Mat-PCK levels of early childhood educators and to examine statistical differences based on various demographic variables. This mixed-methods study utilized the Math-PCK Scale among 50 early childhood educators in Ardahan, Turkey. Sixteen educators representing high and low Mat-PCK levels were identified. These participants then used the Activity Plan Structure Form (APSF) to construct their conceptualizations of mathematics. Quantitative findings indicated that Math-PCK levels varied significantly across educational level, institution type, and the age group of children taught. Qualitative results indicated that educators with high Math-PCK levels demonstrated greater conceptual coherence, developmental appropriateness, and contextualization in their activity plans, while those with low Math-PCK levels presented superficial content with limited pedagogical justification. Overall, the findings highlight the importance of strengthening pedagogical knowledge in early childhood mathematics education by emphasizing that pedagogical content knowledge related to mathematics is shaped not only by educators' formal knowledge but also by how they plan and implement mathematical activities.

### **Introduction**

Early childhood is increasingly recognized as a pivotal step in human development, shaping the foundations of cognitive, emotional, social, and motor development (Bakken et al., 2017). During these years, children exhibit a natural inclination to explore and learn, and when supported by quality educational environments, they rapidly acquire skills that strongly predict later academic success and social adjustment (Howard et al., 2022; Whittaker et al., 2020). Among these areas, early mathematics has received particular attention, as early numerical skills are among the strongest predictors of later school success, often even stronger than early literacy (Aumann et al., 2025; Clements & Sarama, 2015). Consequently, fostering mathematical thinking in early childhood has become a global priority in both research and educational policy (Torbeyns et al., 2024). Children's mathematical knowledge begins to emerge informally long before formal education. Research shows that preschool children can distinguish between sets, recognize shapes, and participate in pattern-building activities, often exceeding the expectations of teachers and curriculum developers (Alsina & Berciano, 2020; Clements & Sarama, 2014). While these informal competencies are universal in their existence, they develop differently depending on cultural, socioeconomic, and pedagogical factors (Gejard & Melander, 2020). Supporting these competencies with opportunities for guided exploration, problem-solving, and reasoning can significantly increase children's engagement and conceptual understanding (Baroody et al., 2019; Papandreou & Tsiouli, 2022). Recent studies also highlight those everyday interactions, such as block play or number talk, provide critical contexts for fostering mathematical reasoning when facilitated by knowledgeable educators (Fuson & Leinwand, 2023; Lundqvist et al., 2023). The roles of teachers are central to shaping these early experiences. Shulman's (1987) concept of Pedagogical Content Knowledge (PCK) emphasizes the intersection of subject knowledge and pedagogy as a unique professional area of expertise for teachers. In mathematics education, PCK refers to educators' abilities to represent mathematical concepts in age-appropriate ways, anticipate misunderstandings, and design tasks that encourage inquiry and conceptual development (Chan & Hume, 2019; Gasteiger et al., 2020). In early childhood, when children's cognitive development is dynamic and diverse, teachers' PABs (Project-Based Learning) are particularly critical in creating inclusive, engaging, and developmentally appropriate math environments (Kim et al., 2024). Despite this importance, research consistently shows that many early childhood educators lack confidence in math instruction and reduce math activities to superficial counting or shape recognition (Orçan-Kaçan & Karayol, 2017; Youmans et al., 2018). Furthermore, while play-based and inquiry-based pedagogies are widely supported, there is no clear consensus on the most effective strategies for fostering sustained math engagement in children (Vogt et al., 2020; Zhang et al., 2024). This uncertainty underscores the need for a deeper investigation into how teachers conceptualize math instruction and the role of their PABs in shaping instructional decisions. In addition, recent international findings highlight that teachers' pedagogical strategies significantly predict not only children's

### **Keywords**

Early Childhood Education  
Pedagogical Content  
Knowledge  
Mathematics Education  
Activity Planning

### **Article Type**

Research Paper

mathematical achievement but also their attitudes toward learning and problem-solving tendencies (Björklund et al., 2020; Willoughby et al., 2021). While early mathematics education is increasingly emphasized in national curricula in Türkiye, the quality of its implementation largely depends on teachers' knowledge, beliefs, and pedagogical orientations (Gökçen & Kutluca, 2022; Karakaş & Kutluca, 2025).

While the importance of early mathematics education is widely acknowledged, the current literature presents several limitations that justify the need for further research. First, many studies focus primarily on pre-service teachers' beliefs, self-efficacy, or general attitudes toward mathematics, rather than their actual Pedagogical Content Knowledge (PCK) and its manifestation in classroom practice (Akdeniz & Şimşek, 2022; Konca & Özçakır, 2021). While these studies provide useful insights into teachers' perceptions, they fall short of examining how educators conceptualize mathematical knowledge and translate it into pedagogically meaningful learning experiences for young children. In other words, there is limited empirical evidence that captures the depth and breadth of teachers' mathematics-specific PCK, particularly in early childhood where developmental appropriateness is critically important. Second, while the international literature is richer in scope, it also exhibits gaps. Most current studies emphasize general principles of early mathematics education such as play-based learning, inquiry, or scaffolding (Björklund et al., 2020; Vogt et al., 2020). However, only a few studies systematically analyze how early childhood educators integrate mathematical content knowledge into pedagogical strategies when planning and implementing classroom activities (Kim et al., 2024; Torbevens et al., 2021). Similarly, while international research confirms the fundamental role of educators' PABs in early mathematics teaching, it also reveals gaps in understanding how this knowledge is reflected in classroom practices (Ginsburg, 2016; Tian & Huang, 2019). Furthermore, cross-cultural comparisons reveal that while informal mathematical skills are universal in children, the degree to which educators utilize these skills depends largely on their pedagogical knowledge and cultural teaching traditions (Alsina & Berciano, 2020; Lundqvist et al., 2023). This situation highlights the need for more context-sensitive research, particularly in non-Western contexts like Turkey, where cultural and curriculum conditions can differ significantly from those in which most international studies are conducted. Third, the Turkish literature on early childhood mathematics education, while growing, is limited in scope and depth. Existing studies have largely focused on pre-service teachers, regional samples, or general attitudes towards mathematics (Sayan, 2023; Yazlık & Öngören, 2018); very few directly examine the PBL (Pedagogical Knowledge) of in-service educators in mathematics. Furthermore, the findings in these studies are often inconsistent: some suggest that pedagogical knowledge increases with experience, while others report little or no correlation between teaching experience and PBL development (Demirbaş, 2019; Lee, 2017; Özdemir, 2020). Such inconsistencies underscore the lack of a comprehensive and systematic understanding of how PBL operates in early childhood mathematics education in Türkiye. Taken together, these gaps highlight the critical need for research that (a) directly examines early childhood educators' math-related PBL (Public-Based Learning), (b) relates their knowledge to how they conceptualize and plan math activities, and (c) addresses the issue with robust methodological approaches. Responding to this need is not only of theoretical importance but also of practical significance for improving professional development programs, informing curriculum design, and ultimately increasing opportunities for young children to engage meaningfully with math. Accordingly, this study aims to investigate early childhood educators' pedagogical content knowledge in mathematics and to examine how this knowledge influences their conceptualizations of planning mathematical activities. To this end, the study seeks to answer the following research problems:

1. What is the level of pedagogical content knowledge regarding mathematics among early childhood educators?
2. Does the pedagogical content knowledge of early childhood educators regarding mathematics differ significantly according to age, educational background, type of institution they work in, and the age group of the children they teach?
3. How does the pedagogical content knowledge of early childhood educators regarding mathematics change their conceptualizations of activity plans?

## Methodology

The aim of this study is to determine the levels of pedagogical content knowledge (PCK) related to mathematics among early childhood educators and to examine how these levels affect their activity planning processes. A mixed-methods design was used to collect quantitative data using the PCK Scale (Dağlı et al., 2019), and the activity plans of early childhood educators with low and high PCK levels were qualitatively analyzed using the Activity Plan Structuring Form (APSF) framework (Kutluca & Mercan, 2022). This section introduces the research design, validity and reliability criteria, participant group, data collection tools, process, and analysis methods.

## Research Design

This study was designed within a mixed-methods framework that combines quantitative and qualitative data collection and analysis to provide more comprehensive answers to research questions (Creswell & Plano Clark,

2017). Specifically, an embedded design was used, where quantitative data were used as the primary element and qualitative data were included to enrich and explain the findings (Creswell, 2021). The rationale for this choice lies in the study's dual objective: to quantitatively measure early childhood educators' levels of pedagogical content knowledge (PCK) related to mathematics and simultaneously investigate how these levels are reflected in pedagogical planning processes; this is an aspect that cannot be fully captured with numerical data alone. In practice, the PCK levels of 50 participants were first assessed using a standardized scale. Based on their scores, educators from both upper and lower groups were selected to provide qualitative data. This data consisted of mathematics activity plans analyzed through APSF (Kutluca & Mercan, 2022), allowing for an in-depth examination of the components of pedagogical knowledge. This multi-layered analysis aimed to relate educators' Math-PCK levels to how they conceptualize and design mathematics activities. Accordingly, embedded mixed methods design provided a functional framework for both defining Math-PCK levels and interpreting their impact on classroom practices (Ivankova et al., 2006; Johnson & Onwuegbuzie, 2004).

**Ensuring Validity and Reliability Criteria.** Ensuring Validity and Reliability Criteria. In this study, validity and reliability issues were addressed in accordance with both quantitative and qualitative research principles. In the quantitative phase, the Math-PCK Scale (Dağlı et al., 2019) was used. The original development work ensured content validity through expert review and construct validity through confirmatory factor analysis. Reliability was confirmed with a Cronbach's alpha value above .80, an indicator of satisfactory internal consistency. In this research, the scale was administered directly, and scoring procedures were performed according to standardized guidelines. Criterion-based sampling was used to ensure internal validity in the selection of lower and upper groups, while the representation of both public and private institutions in the research area supported external validity (Patton, 2014). In the qualitative phase, data were analyzed using inductive content analysis procedures (Elo & Kyngäs, 2008). Triangulation was achieved by analyzing both written activity plans and accompanying oral descriptions, and direct quotations were used to enhance reliability (Creswell & Poth, 2016). Inter-coder reliability was ensured by having a second expert independently code a subset of the data and by calculating agreement using Miles and Huberman's (1994) formula. Consistency was further ensured through continuous comparison (Glaser & Strauss, 2017). Peer review and documentation of all analytical steps increased transparency and reliability (Lincoln & Guba, 1985). Overall, validity and reliability were systematically addressed in both quantitative and qualitative phases to ensure that the data provided reliable and meaningful answers to the research questions.

## Participants

The participants in this study consisted of 50 early childhood educators working in independent preschools affiliated with the Ministry of National Education (MEB) in Ardahan during the 2024-2025 academic year. Participants were selected through criterion sampling, a purposeful strategy involving selecting individuals who directly experience the phenomenon under investigation and can provide in-depth knowledge (Patton, 2014). Inclusion criteria were: (1) being an active preschool teacher in an independent preschool or pre-school unit affiliated with the MEB and (2) holding at least an associate degree in child development or a bachelor's degree in early childhood education. These criteria were chosen to ensure that participants possessed both theoretical knowledge and practical experience related to Pedagogical Content Knowledge (PCK) in mathematics (Creswell & Poth, 2016). Thus, the sample consisted of professionals actively involved in planning and implementing early mathematics activities, enabling the examination of the interaction between pedagogical knowledge and practice. To enhance the representation of the participating group, diversity was also ensured in terms of institution type, educational background, and age groups taught (Yildirim & Simsek, 2021). Detailed demographic information is presented in Table 1.

Table 1  
Participants

| Institution Type | Education Level   | Age Group Worked With |              |              | Total     |
|------------------|-------------------|-----------------------|--------------|--------------|-----------|
|                  |                   | 36-48 months          | 48-60 months | 60-72 months |           |
| Private          | Associate Degree  | 12                    | 2            | 0            | 14        |
|                  | Bachelor's Degree | 11                    | 3            | 2            | 16        |
|                  | <b>Total</b>      | <b>23</b>             | <b>5</b>     | <b>2</b>     | <b>30</b> |
| Public           | Associate Degree  | 1                     | 0            | 0            | 1         |
|                  | Bachelor's Degree | 10                    | 6            | 3            | 19        |
|                  | <b>Total</b>      | <b>11</b>             | <b>6</b>     | <b>3</b>     | <b>20</b> |

|                    |                   |           |           |          |           |
|--------------------|-------------------|-----------|-----------|----------|-----------|
|                    | Associate Degree  | 13        | 2         | 0        | 15        |
| <b>Grand Total</b> | Bachelor's Degree | 21        | 9         | 5        | 35        |
|                    | <b>Total</b>      | <b>34</b> | <b>11</b> | <b>5</b> | <b>50</b> |

Table 1 shows the distribution of the 50 participating early childhood educators according to institution type, educational background, and age group they teach. 60% of the sample is from private institutions and 40% from public institutions. In terms of educational level, 35 participants (70%) hold a bachelor's degree and 15 participants (30%) hold an associate's degree. Most associate's degree holders in private institutions work with the 36-48 month age group, while all educators teaching the 60-72 month age group hold a bachelor's degree. In public institutions, almost all educators (n = 19) hold a bachelor's degree, and there is a more balanced distribution across age groups.

**Determining the Subsample.** Determining the Subsample. To examine how early childhood educators' Math-PCK levels affect their conceptualization of mathematics activities, participants were divided into subgroups and subgroups based on their scores on the Math-PCK Scale. This process utilized extreme case sampling, a purposeful strategy that selects participants at both ends of a distribution to provide clearer contrasts and richer insights into the phenomenon (Patton, 2014; Yıldırım & Şimşek, 2021). Initially, scores were calculated for all 50 educators, and descriptive statistics (mean and standard deviation) were used to determine their endpoints. Educators scoring above the mean plus half a standard deviation were categorized as the high Math-PCK group, while those scoring below the mean minus half a standard deviation were assigned to the low Math-PCK group. This systematic approach, often applied in qualitative research to increase depth of knowledge, allowed for meaningful comparisons between educators with opposing levels of pedagogical content knowledge (Palinkas et al., 2015). Detailed information about these groups is provided in Table 2.

Table 2  
*Information about Participants in the Subsample*

|                 | Participant | Education         | Experience         | Institution Type | Age Group Worked With | Math-PCK Score |
|-----------------|-------------|-------------------|--------------------|------------------|-----------------------|----------------|
| Math-PCK (Low)  | <b>D-1</b>  | Bachelor's degree | 0-5 years          | Private          | 60-72 months          | 12             |
|                 | <b>D-2</b>  | Associate degree  | 0-5 years          | Private          | 60-72 months          | 13             |
|                 | <b>D-3</b>  | Bachelor's degree | 0-5 years          | Private          | 48-60 months          | 13             |
|                 | <b>D-4</b>  | Associate degree  | 0-5 years          | Private          | 36-48 months          | 14             |
|                 | <b>D-5</b>  | Associate degree  | 0-5 years          | Private          | 48-60 months          | 15             |
|                 | <b>D-6</b>  | Associate degree  | 0-5 years          | Private          | 48-60 months          | 15             |
|                 | <b>D-7</b>  | Associate degree  | 6-10 years         | Private          | 48-60 months          | 16             |
|                 | <b>D-8</b>  | Bachelor's degree | 0-5 years          | Public           | 48-60 months          | 16             |
| Math-PCK (High) | <b>Y-1</b>  | Bachelor's degree | 0-5 years          | Private          | 36-48 months          | 30             |
|                 | <b>Y-2</b>  | Bachelor's degree | 6-10 years         | Private          | 60-72 months          | 30             |
|                 | <b>Y-3</b>  | Associate degree  | 0-5 years          | Private          | 48-60 months          | 31             |
|                 | <b>Y-4</b>  | Bachelor's degree | 0-5 years          | Public           | 36-48 months          | 31             |
|                 | <b>Y-5</b>  | Bachelor's degree | 0-5 years          | Private          | 60-72 months          | 32             |
|                 | <b>Y-6</b>  | Bachelor's degree | 0-5 years          | Public           | 48-60 months          | 32             |
|                 | <b>Y-7</b>  | Bachelor's degree | 16 years and above | Public           | 48-60 months          | 32             |
|                 | <b>Y-8</b>  | Bachelor's degree | 0-5 years          | Public           | 60-72 months          | 34             |

As shown in Table 2, the low Math-PCK group consisted of eight educators, five with associate's degrees and three with bachelor's degrees, all with 0-10 years of teaching experience. Most had worked in private institutions (n = 7) and taught children aged 48-60 months. In contrast, the high Math-PCK group consisted of eight educators, seven with bachelor's degrees and one with an associate's degree. Although most had 0-5 years of experience, the group also included educators with longer professional seniority (6-10 years and 16+ years). These participants were more evenly distributed between public and private institutions and taught groups across a wider age range (36-72 months). Their Math-PCK scores ranged from 30 to 34, indicating relatively strong pedagogical content knowledge in mathematics.

## Data Collection Tools

Two different data collection tools were used to answer the sub-problems of this research. The Math-PCK Scale, developed by Dağlı et al. (2019), was used to determine the level of pedagogical content knowledge of early childhood educators regarding mathematics and to divide them into two subgroups. On the other hand, the Activity Plan Structuring Form (APS), developed by Loughran et al. (2004) and adapted by Kutluca and Mercan (2022) to Turkish and early childhood education, was used to compare the mathematics activity plans of early childhood educators with low and high levels of pedagogical content knowledge regarding mathematics. These data collection tools are introduced in detail below.

**Pedagogical Content Knowledge Scale regarding Mathematics (Math-PCK).** To assess the pedagogical content knowledge of early childhood educators in the field of mathematics, the Preschool Teachers' Mathematics Pedagogical Content Knowledge Scale developed by Dağlı et al. (2019) was used. The scale is scenario-based and designed to evaluate teachers' ability to analyze children's mathematical expressions during play, thus providing a multidimensional measure of pedagogical content knowledge. It consists of five scenarios, each with seven items, for a total of 35 items. The items cover both mathematical content domains (e.g., counting, geometry, spatial relationships, part-whole, matching, grouping, comparing, measuring, operations, patterns, graphs) and process domains (problem solving, reasoning, connecting, symbolizing, communication) (Dağlı et al., 2019; NCTM, 2000). Each item allows for more than one correct answer. Scoring is done on a weighted basis where correct answers are added to the score and incorrect answers are subtracted (Frary, 1989). This approach measures educators' mathematical knowledge as well as their analytical reasoning and decision-making skills, reflecting their ability to resolve misunderstandings. The normative mean score of the scale was reported as 17.5.

**Activity Plan Structuring Form.** In the qualitative phase of the research, the Activity Plan Structuring Form (APSF) was used to examine how early childhood educators' Math-PCK levels are reflected in their activity planning. Adapted by Kutluca and Mercan (2022) from Loughran et al.'s (2004) original content representation framework to the Turkish early childhood context, the APSF provides a structured format for educators to express what, why, and how they will teach a particular mathematical concept. The tool encourages educators to identify core ideas, intended learning outcomes, process skills, potential misunderstandings, and teaching strategies, revealing the pedagogical reasoning that underlies their planning (Clements & Sarama, 2020; Nilsson, 2014). The APSF conceptualizes teachers' knowledge beyond declarative content and emphasizes the integration of subject matter and pedagogy as it emerges in real teaching contexts (Kind & Chan, 2019). Information regarding the characteristics of the eight questions included in the APSF is given in Table 3.

Table 3  
*Characteristics of APSF Questions*

| Question Content  | Dimension/Characteristic it Represents                  |
|---|---|
| 1. What do you aim for children to learn through this activity?   | <i>Instructional Objectives and Outcomes</i>            |
| 2. Why is it important for children to be familiar with the topic or theme associated with this activity? | <i>Value and Justification of the Content</i>           |
| 3. What other information do you know that children don't necessarily need to know?                       | <i>Subject Matter Knowledge and Content Selection</i>   |
| 4. What challenges/limitations will you encounter while conducting this activity?                         | <i>Instructional Challenges and Limitations</i>         |
| 5. What children's thoughts/concepts influenced your decision to conduct the activity?                    | <i>Learner Knowledge</i>                                |
| 6. What types of teaching approaches will you use during this activity?                                   | <i>Teaching Methods and Strategies</i>                  |
| 7. How will you determine if children have correctly understood the topic covered in the activity?        | <i>Assessment and Observation</i>                       |
| 8. What resources did you use to prepare for the activity?  | <i>Teacher Preparation and Professional Development</i> |

To ensure content validity, the form was reviewed by early childhood mathematics education experts and revised for clarity and relevance. A pilot study with two educators not included in the main sample further supported its usability. During data collection, participants completed the APSF through individual sessions, providing both

written responses and recorded and matched oral statements. On average, each session lasted 60 minutes. This process provided an in-depth qualitative analysis of how educators' Math-PCKs influenced their instructional decisions.

### Data Collection Process

Data collection was carried out in three steps (Figure 1). In the first step, participants were informed about the ethical principles and the general purpose of the study, and 50 volunteer early childhood educators from preschools affiliated with the Ministry of National Education in Ardahan were included in the study. The Mathematics Pedagogical Content Knowledge Scale (Daglı et al., 2019) was administered through face-to-face sessions, each lasting approximately 20-25 minutes, with guarantees of confidentiality, voluntary participation, and the right to withdraw. In the second step, the scores were analyzed, and participants were divided into high and low Math-PCK groups using mean  $\pm 0.5$  SD as cutoff points. In this process, 16 educators (8 low, 8 high) were selected for the qualitative phase.

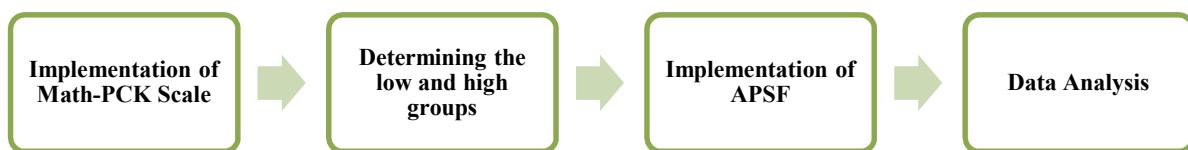


Figure 1. Data Collection Process

In the third step, these participants completed the APSF, where they selected a mathematical concept and summarized its core ideas, learning outcomes, and process skills. Each session lasted approximately 60 minutes, and verbal explanations were audio-recorded with permission to support subsequent content analysis. Transparency, ethical sensitivity, and participant safety were prioritized throughout all stages. Data collection was conducted in comfortable environments chosen by the participants, interviews were uninterrupted, and all data was kept confidential and securely stored until analysis was complete.

### Data Analysis

Due to the nature of the research questions, a mixed methods approach was adopted, and analyses were conducted on two axes: quantitative and qualitative. Quantitative analyses were applied to the first two sub-questions, while qualitative analyses were applied to the third sub-question. Responses to the Math-PCK Scale were first examined for normality using the Kolmogorov-Smirnov test, which confirms normal distribution (Büyüköztürk, 2022). Then, descriptive statistics (mean, SD, lowest-highest) were used to determine general knowledge levels (Fraenkel et al., 2012). One-way ANOVA was used for multi-level demographic variables such as age groups to test for group differences, and independent samples t-tests were applied for binary variables such as institution type and degree level (Pallant, 2020). All analyses were performed in SPSS with a significance level of .05. For the qualitative phase, inductive content analysis (Mayring, 2021) was conducted to compare how early childhood educators with high and low levels of Math-PCK conceptualize mathematical activities. In this analysis process, written and verbal data collected from participants via APSF were systematically examined according to the continuous comparative method. In this context, all documents were carefully read several times before coding and recurring meaningful statements serving the research questions were identified. During the analysis, meaning units were defined and converted into explicit codes; similar codes were grouped to form concepts, and these concepts were abstracted into higher-level themes (Selvi, 2019). This structuring process was used to reveal which pedagogical tendencies educators emphasized in their planning, how they conceptualized mathematical ideas, and which learning objectives they prioritized. In determining the themes, structures frequently repeated in the participants' statements, emphasized cause-and-effect relationships, and pedagogical justifications were considered. In addition, participants' questions such as "Why did I choose this learning outcome?", "What do I expect the child to learn?", and "Which concepts might be challenging?" were also considered. Responses to questions such as these were decisive in the creation of themes (Tisdell et al., 2025). Reliability was ensured by coding a subset of the data twice; inter-coder agreement was 91%, exceeding the 80% threshold recommended for qualitative research (Miles & Huberman, 1994). Themes were presented in tables and shown with direct quotations, increasing both reliability and depth (Tisdell et al., 2025). This structured process allowed for a reliable comparison of the pedagogical conceptualizations of educators with high and low Math-PCK, providing both statistical and interpretive insights into their approaches to planning mathematics activities.

## Findings

This section first assesses the normality of participants' Math-PCK scores using the Kolmogorov-Smirnov test. Then, while descriptive statistics are reported to address the first sub-problem, independent samples t-tests and one-way ANOVA were applied to examine differences based on age, educational background, type of institution, and age group taught. Finally, inductive content analysis of qualitative data compared the teaching approaches and conceptualizations of high- and low-level Math-PCK educators. The findings are systematically organized under subheadings supported by tables and direct quotations.

Table 4  
*Kolmogorov-Smirnov Normality Test Results*

| Math-PCK | N  | $\bar{X}$ | Standard deviation | p    |
|----------|----|-----------|--------------------|------|
|          | 50 | 22,30     | 5,68               | ,750 |

$p > .05$

Before proceeding with the analysis of quantitative data, the distribution characteristics of the main variable, Math Pedagogical Content Knowledge (Math-PCK) scores, were tested according to the assumptions of parametric analyses using the Kolmogorov-Smirnov normality test. The results in Table 4 showed that the scores were normally distributed ( $p = .750$ ) because the p-value exceeded the significance threshold of .05. Accordingly, it was assessed that parametric statistical methods were suitable for subsequent analyses. Descriptive statistics and group comparisons addressing the sub-research questions were conducted under this assumption, and the findings are presented systematically in tables.

### Math-PCK Levels of Early Childhood Educators

The findings of the descriptive statistical analysis conducted on the responses of early childhood educators participating in the study to the Math-PCK Pedagogical Content Knowledge Scale are presented in Table 5. The normative mean for the Math-PCK Scale is 17.5.

Table 5  
*Descriptive Statistics Results*

| Math-PCK | N  | Minimum | Maximum | $\bar{X}$ | Standard deviation |
|----------|----|---------|---------|-----------|--------------------|
|          | 50 | 12      | 34      | 22,30     | 5,676              |

According to the findings in Table 5, participants' Math-PCK scale scores ranged from 12 to 34, with an arithmetic mean of 22.30 and a standard deviation of 5.676. This result indicates that the vast majority of early childhood educators did not reach the maximum score on the scale, but their performance levels were significantly above the scale's normative mean of 17.5. In other words, while the pedagogical content knowledge levels of early childhood educators in the research group were higher than the scale's accepted standard mean, they remained at a moderate level compared to the ideal level (35 full points). On the other hand, the relatively high standard deviation of 5.676 suggests significant variation in participants' pedagogical content knowledge levels. This indicates high individual differences in pedagogical knowledge among educators and limited intergroup homogeneity. It is thought that these differences may vary according to the educational levels of early childhood educators, the type of institution they work in, or their age groups. Therefore, in the following analyses, the effect of these demographic variables on Math-PCK was examined, and whether these differences were statistically significant was tested.

To determine the statistical significance of the variation of Math-PCK scores, which represent the pedagogical content knowledge of participating early childhood educators and meet the assumption of normal distribution, based on age and the child's age group, a One-Way Analysis of Variance (ANOVA) was conducted. To determine the variation based on educational status and type of institution worked in, an independent samples t-test was performed on the data. The findings obtained from these tests are presented using different tables for each test. At this stage, the descriptive statistics and ANOVA results obtained after the analyses conducted to determine the variation of the average Math-PCK scores of the participating early childhood educators according to the age variable are presented in Table 6 and Table 7, respectively.

Table 6  
*Descriptive Statistics of Math-PCK Scores by Age*

| Age          | N         | $\bar{X}$    | S.D.         |
|--------------|-----------|--------------|--------------|
| 20-25        | 6         | 19,50        | 4,593        |
| 26-30        | 21        | 21,14        | 5,313        |
| 31-35        | 13        | 23,31        | 6,223        |
| 36 and above | 10        | 25,10        | 5,547        |
| <b>Total</b> | <b>50</b> | <b>22,30</b> | <b>5,676</b> |

As shown in Table 6, the mean Math-PCK scores of participants aged 20-25 were 19.50, those aged 26-30 were 21.14, those aged 31-35 were 23.31, and educators aged 36 and over were 25.10. Considering the overall mean of 22.30, the findings indicate an increasing trend in Math-PCK levels with age. In particular, the scores of educators aged 36 and over are significantly above the normative mean of 17.5, suggesting that age and professional experience contribute positively not only to practical expertise but also to pedagogical knowledge in mathematics. The statistical significance of these differences was further tested using ANOVA, as presented in Table 7.

Table 7  
*Age-Related ANOVA Results of Math-PCK Scores*

| Source of Variance | Sum of Squares | df | Mean Square | F     | p    | Significant Difference |
|--------------------|----------------|----|-------------|-------|------|------------------------|
| Between Groups     | 166.759        | 3  | 55.586      | 1.811 | .158 | None                   |
| Within Groups      | 1411.741       | 46 | 30.690      |       |      |                        |
| Total              | 1578.500       | 49 |             |       |      |                        |

According to the analysis results, the sum of squares between groups was found to be 166.759 and the sum of squares within groups was 1411.741. The obtained F value is 1.811 and the significance level is p=.158. Since this p value is greater than the traditional threshold of .05, no statistically significant difference was found between age groups in terms of Math-PCK scores [ $F(3, 46) = 1.82, p > .05$ ]. This result shows that although descriptive statistics show an increasing trend in Math-PCK as age increases, the differences are not statistically significant. The results of the One-Way Analysis of Variance (ANOVA) conducted to determine whether the mean Math-PCK scores of the early childhood educators participating in the study differed significantly according to the child's age group variable are given in Tables 8 and 9.

Table 8  
*Descriptive Statistics of Math-PCK Scores According to Child's Age Group*

| Child's Age Group | N         | $\bar{X}$    | S.D.         |
|-------------------|-----------|--------------|--------------|
| 36-48 ay          | 34        | 20,94        | 5,354        |
| 48-60 ay          | 11        | 24,55        | 5,956        |
| 60-72 ay          | 5         | 25,40        | 4,393        |
| <b>Total</b>      | <b>50</b> | <b>22,30</b> | <b>5,676</b> |

As shown in Table 8, descriptive statistics illustrate the mean and standard deviation of Math-PCK scores across different age groups of children. Educators teaching children aged 36-48 months had the lowest mean score (Mean = 21), while those teaching the 60-72-month age group achieved the highest mean score (Mean = 25.4). These findings suggest that educators working with older children tend to demonstrate stronger pedagogical content knowledge related to mathematics. The statistical significance of these differences was examined using ANOVA, and the results are presented in Table 9.

Table 9  
*ANOVA Results of Math-PCK Scores According to Child's Age Group*

| Source of Variance | Sum of Squares | df | Mean Square | F     | p    | Significant Difference |
|--------------------|----------------|----|-------------|-------|------|------------------------|
| Between Groups     | 200,690        | 2  | 100,345     | 3,423 | ,041 | <i>36-48 months*</i>   |
| Within Groups      | 1377,810       | 47 | 29,315      |       |      | <i>48-60 months</i>    |
| Total              | 1578,500       | 49 |             |       |      | <i>60-72 months</i>    |

According to the analysis in Table 9, Math-PCK scores differed significantly among child age groups [ $F(2, 47) = 3.42, p < .05$ ]. Descriptive means showed that the highest scores belonged to educators teaching children aged 60-72 months, while the lowest scores were observed among educators teaching children aged 36-48 months. Subsequent comparisons confirmed that educators working with older age groups had significantly higher levels of pedagogical content knowledge related to mathematics. To determine the statistical significance of the variation in Math-PCK scores, which represent the pedagogical content knowledge levels of early childhood educators participating in the study and are within the parameters of a normal distribution, according to educational status, an independent samples t-test was performed on the data. The t-test results are given in Table 10.

Table 10  
*Results of the t-test for Math-PCK Mean Scores According to Educational Status*

| Educational Status | N  | $\bar{X}$ | Standard Deviation | df | t      | P    |
|--------------------|----|-----------|--------------------|----|--------|------|
| Associate degree   | 15 | 19,27     | 4,949              | 48 | -2,618 | ,012 |
| Bachelor's Degree  | 35 | 23,60     | 5,526              |    |        |      |

*p*<,.05

According to the t-test results presented in Table 10, the pedagogical content knowledge of early childhood educators regarding mathematics differed significantly according to their educational status [ $t(48) = -2.62, p < .05$ ]. In particular, those with a bachelor's degree ( $X = 23.6$ ) scored significantly higher than those with an associate degree ( $X = 19.3$ ). This finding revealed a significant relationship between pedagogical content knowledge of mathematics and the educational status variable, favoring the participants with a bachelor's degree. Whether the Math-PCK scores of early childhood educators differed significantly according to the type of institution they worked in was analyzed through independent samples t-test. The t-test results are given in Table 11.

Table 11  
*Results of the t-test for Math-PCK Average Scores by Institution Type*

| Institution Type | N  | $\bar{X}$ | Standard Deviation | df | t      | P    |
|------------------|----|-----------|--------------------|----|--------|------|
| Private          | 30 | 20,40     | 5,697              | 48 | -3,152 | ,003 |
| Public           | 20 | 25,15     | 4,392              |    |        |      |

*p*<,.05

According to the t-test results presented in Table 11, the pedagogical content knowledge of early childhood educators regarding mathematics differed significantly according to the type of institution [ $t(48) = -3.15, p < .05$ ]. Educators working in public schools ( $X = 25.2$ ) scored significantly higher than educators in private schools ( $X = 20.4$ ). This finding revealed a significant relationship between pedagogical content knowledge regarding mathematics and the type of institution worked in favor of the participants working in public schools.

### The Impact of Math-PCK Level on Activity Plan Conceptualizations

An inductive content analysis was performed on the conceptualizations of the eight early childhood educators with the highest Math-PCK scores and the eight early childhood educators with the lowest scores, based on their responses to the APSF. Both groups identified a topic in their activity plans, along with Big Idea-1 and Big Idea-2. The findings showed significant conceptual and pedagogical differences between the high and low Math-PCK groups (Table 12).

Table 12  
*Topic and Big Ideas Regarding Math Activities*

| Math-PCK Level | Topic   | Big Ideas   |
|----------------|---|---|
| Low Math-PCK   | D-1 Performing simple addition and subtraction using concrete objects | 1. Performing simple addition and subtraction using objects and applying them to daily life situations.<br>2. Increasing or decreasing a group by a specified number of objects through addition and subtraction.                         |
|                | D-2 Addition  | 1. Performing simple addition using concrete objects.<br>2. Performing addition without using objects.  |
|                | D-3 Geometric shapes  | 1. Helping children understand geometric shapes.<br>2. Teaching geometric shapes through games and songs.   |
|                | D-4 Square  | 1. Teaching the shape “square” and reinforcing learning through art activities and games to ensure retention.   |
|                | D-5 Whole, half, quarter  | 1. Understanding fractions at the kindergarten level through real-life activities.<br>2. Using acquired knowledge in daily life.  |
|                | D-6 Learning about money  | 1. Recognizing money.<br>2. Ordering money.   |
|                | D-7 Let's catch the colorful fish                                     | 1. Presenting basic mathematical concepts appropriate to children's developmental levels.<br>2. Learning mathematical concepts through active participation with concrete materials in interactive learning environments.                 |
|                | D-8 Rhythmic counting   | 1. Developing rhythmic counting skills by placing train wagons sequentially.<br>2. Reinforcing number-object correspondence by adding the specified number of wagons.   |
| High Math-PCK  | Y-1 Recognizing numbers from 1 to 10                                  | 1. Learning quantities and numbers by counting objects.<br>2. Developing mathematical skills through counting.  |
|                | Y-2 Numerals  | 1. Recognizing numerals and using them in daily life.<br>2. Learning through play, concretization, and learning by doing; developing a love for mathematics.  |
|                | Y-3 Learning addition   | 1. Learning to increase and add quantities.<br>2. Combining objects used in daily life, such as fruits.   |
|                | Y-4 Let's count on colorful blocks                                    | 1. Understanding basic mathematical concepts appropriate to developmental levels.<br>2. Learning mathematical concepts through concrete materials and active participation in social learning environments.                               |
|                | Y-5 Subtraction   | 1. Developing forward and backward rhythmic counting; supporting sequencing, matching, classification, and problem solving.<br>2. Reinforcing addition and subtraction using concrete materials; emphasizing grouping and sharing skills. |
|                | Y-6 Learning the number 7   | 1. Recognizing and solving daily life problems; developing early literacy skills.<br>2. Rhythmic counting between 1 and 20 and establishing one-to-one correspondence.  |
|                | Y-7 One-to-one correspondence   | 1. Supporting meaningful counting skills.<br>2. Encouraging counting by touching objects and matching numbers with objects.   |
|                | Y-8 Numbers   | 1. Recognizing numbers and symbols and using them in daily life.<br>2. Establishing cause-effect relationships.   |

An examination of the responses of early childhood educators with high Math-PCK levels revealed that their instructional conceptualizations were characterized by consistency, developmental congruence, clear reasoning, and pedagogical depth integrated with children's life experiences. Rather than limiting themselves to identifying a mathematical topic, these educators explained why the concept should be addressed, what competencies children were expected to develop, and how the instructional process could be adapted to developmental needs. For example, statements such as, "Since addition and subtraction are abstract concepts, I use concrete materials to reinforce children's understanding of these operations," exemplify a reflective approach that integrates subject knowledge with pedagogical strategy. This type of reasoning demonstrates an advanced ability to integrate content and pedagogy in ways that mirror Shulman's (1987) framework of pedagogical content knowledge. In contrast, the responses of educators in the low Math-PCK group revealed limited pedagogical depth. Their explanations were often limited to superficial descriptions, activity demonstrations, or narrowly defined behavioral consequences, and offered little or no justification for instructional choices. Responses such as "teach the square," "count objects," or "introduce coins" showed a tendency to focus directly on the content without grounding it in a broader pedagogical rationale. Similarly, instructions such as "remove or add objects from a set according to a specific number" reflected procedural thinking rather than consideration of how such tasks might support children's conceptual understanding or why the activity might be pedagogically significant. These patterns suggest that educators with lower levels of Math-PCK approach instructional planning in an activity-oriented way, placing less emphasis on the underlying developmental and conceptual dimensions of learning. A comparative analysis between the two groups highlighted "developmental appropriateness" as a central differentiating theme. Higher Math-PCK educators selected content and designed activities consistent with children's cognitive capacities and learning needs. Analysis of responses from educators in the lower Math-PCK group revealed that most did not explicitly refer to the concept of developmental appropriateness, and activity descriptions remained largely superficial. The second theme that emerged from the comparative analysis was conceptual connections. Educators in the high Math-PCK group systematically made meaningful connections between mathematical concepts, such as grouping, matching, and the relationship between addition and subtraction, while such integrative reasoning was rarely observed in the responses of those in the low Math-PCK group. Another prominent theme was contextualization through everyday experiences. High Math-PCK participants frequently emphasized the importance of placing mathematical learning in real-life contexts and designing activities that reflect children's natural encounters and daily routines. In contrast, educators with low Math-PCK levels tended to frame mathematics instruction in abstract and decontextualized ways, providing children with limited opportunities to relate mathematical concepts to their life experiences. Additionally, educators with high Math-PCK levels consistently justified teaching decisions with pedagogical reasoning, whereas such explanations were largely absent in the responses of the low group. This contrast demonstrates that educators with higher pedagogical content knowledge approach mathematics activity planning not merely as content presentation, but as a process integrating developmental needs, contextual learning opportunities, and conscious pedagogical structuring. Conversely, educators with lower levels of Math-PCK, with limited pedagogical depth, primarily relied on superficial content presentation. The concepts and themes that emerged from the inductive content analysis are presented in Table 13.

Table 13  
*Themes and Concepts Related to Inductive Content Analysis*

| High Math-PCK Group  | Low Math-PCK Group   |
|--|--|
| <b>Developmental Appropriateness</b><br>Age-appropriate concept selection<br>Concretization of abstract concepts               | <b>Content Orientation</b><br>Goal being solely content transmission<br>No explanation regarding concept level                           |
| <b>Conceptual Depth</b><br>One-to-one correspondence, rhythmic counting<br>Conceptual approach to operation concepts           | <b>Conceptual Superficiality</b><br>Number recognition, shape recognition<br>Presenting operations only as result-oriented               |
| <b>Problem Solving and Reasoning</b><br>Estimation, inference, cause-effect relationship<br>Creating scenarios from daily life | <b>Lack of Problem Posing</b><br>Absence of problem scenarios  |
| <b>Contextual and Experiential Learning</b><br>Associating with daily life<br>Emphasis on the functionality of learning        | <b>Decontextualized Learning</b><br>Abstract and decontextualized explanations   |
| <b>Multiple Developmental Domains</b><br>Social-emotional development<br>Motor skills / Language use                           | <b>One-Dimensional Development Emphasis</b><br>Explanations limited to cognitive processes<br>Not addressing other developmental domains |
| <b>Justification and Pedagogical Grounding</b><br>Why this concept?<br>How is it learned better?                               | <b>Lack of Justification</b><br>Unjustified explanations like "I just want to teach it"  |

|    |   |  |
|----|---|--|
| Q2 | <b>Meaningful Understanding and Internalization</b><br>Meaning of learning the concept for the child<br><b>Functional Link with Daily Life</b><br>Integrating math with life<br><b>Concept Principles and Cognitive Correctness</b><br>One-to-one correspondence, meaningful counting principles<br><b>Holistic Development</b><br>Attention, thinking, language development, early literacy  | <b>Basic Concept Recognition and Content Knowledge</b><br>Superficial emphasis on where to use concepts in daily life<br><b>Symbolic Link with Daily Life</b><br>Simple contexts like shopping with money, slicing cake<br><b>Limited Mention of Social-Emotional Development</b><br>Emphasis on individual skills like creativity, problem solving  |
|    | <b>Awareness of Conceptual Abstraction</b><br>Negative numbers<br>Prime numbers<br>Number line<br>Sets<br><b>Sensitivity to Developmental Limits</b><br>Pre-operational stage<br>Concrete operations<br>Intuitive learning  | <b>Information Simplification</b><br>Degree concept<br>Area and diameter info<br>Country currencies<br>Multi-step operations<br><b>Age-Based Exclusion</b><br>“Not suitable for 3–4 years” approach<br>Developmental stage rationale<br>Postponing transfer based on age<br><b>Practice-Based Exclusion</b><br>Rationale that measurement is not done<br>No need for symbolic representation<br>Exclusion because content is abstract<br><b>Emphasis on Future Knowledge</b><br>“Will learn in the future” expression<br>Deferred content for the child<br><b>Unjustified Information Exclusion</b><br>“I didn’t include it” expression<br>Not specifying exclusion rationale<br>Lack of awareness regarding pedagogical quality of information domain |
|    | <b>Pedagogical Justified Filtering</b><br>Filtering due to learning constraints<br>Activity design-oriented limitation<br>Conceptual density adjustment<br><b>Implicit Goal Awareness</b><br>Unconscious concept acquisition<br>Non-objective intuitive transfer<br><b>Interdisciplinary Cognitive Sensitivity</b><br>Math-thinking relationship<br>Number-logic link<br>Establishing links between concepts  |  |
|    | <b>Cognitive Development Differences</b><br>Lack of readiness<br>Insufficient prior knowledge<br>Confusing concepts<br><b>Conceptual Abstraction and Meaning-Making</b><br>Number, quantity, symbol difficulties<br>One-to-one correspondence issues<br>Memorization – meaning-making distinction<br><b>Attention and Interest Management</b><br>Short attention span<br>Interest fading quickly<br>Weakening motivation<br><b>Material and Environmental Conditions</b><br>Insufficient tools/materials<br>Suitability to environment, need for adaptation<br><b>Instructional Adaptation and Differentiation</b><br>Adaptation for physical disability<br>Enrichment based on group level<br>Flexibility in teaching strategy | <b>Cognitive Development Differences</b><br>Difficulty in number-quality matching<br>Differences in perception level<br>Rhythmic counting difficulty<br><b>Conceptual Confusion</b><br>Distinguishing addition-subtraction<br>Geometric shape recognition<br>Understanding value of money<br><b>Interest-Attention Deficit</b><br>Resistance to subtraction<br>Concepts taking time<br><b>Lack of Materials (Implicit)</b><br>Cannot make meaning without visual support (indirect)<br><b>Reactive Approach to Differences</b><br>Need for extra individual work<br>Inability to achieve synchronous in-class learning   |

|            |  |  |
|------------|--|--|
| <b>Q5</b>  | <b>Awareness of Developmental Limitations</b>                | <b>Age and Development Differences</b>                 |
|            | Concrete-abstract transition difficulty                      | Simplification for 3-4 years                           |
|            | Need for conceptual effort                                   | Narrative change according to different age groups     |
|            | <b>Detection of Prior Knowledge and Misconceptions</b>       | <b>Misconcept Difference (Limited)</b>                 |
|            | Confusing concepts   | Square drawing – circle drawing distinction            |
| <b>Q6</b>  | Difficulty understanding symbols                             | Inability to count rhythmically and one-to-one         |
|            | Negative transfers (contradiction with home-based knowledge) | Perceptually confusing value of money                  |
|            | <b>Child Ideas Enriching Presentation</b>                    | <b>Participant Child Contribution (Unclear)</b>        |
|            | Asking questions and curiosity                               | Drawing symbols in free activity                       |
|            | Experience sharing   | “Bright idea” expression (general, no example)         |
| <b>Q7</b>  | Alternative thinking styles                                  | <b>Time and Attention Limitation</b>                   |
|            | <b>Need for Flexibility in Teaching Strategies</b>           | Shortening of activity duration                        |
|            | Adapting plan to child thoughts                              | Some children showing disinterest                      |
|            | Being prepared for distraction                               |  |
|            |  |  |
| <b>Q6</b>  | <b>Play and Interaction-Based Education</b>                  | <b>Concretization and Applied Education</b>            |
|            | Learning through play  | Object-based demonstration                             |
|            | Drama and imitation  | Show and do  |
|            | Active participation   | Learning by doing                                      |
|            | <b>Education Sensitive to Learning Process</b>               | <b>Sensitivity to Individual Differences (Limited)</b> |
| <b>Q7</b>  | Developmental interaction approach                           | Adapting narrative to class level                      |
|            | Individualization  | Creating student-centered environment                  |
|            | <b>Multiple Education Strategies</b>                         | <b>Singular and Superficial Strategy Use</b>           |
|            | Discovery learning   | Verbal narration, visual support                       |
|            | Speaking ring, fishbowl technique                            | Worksheets, homework                                   |
| <b>Q8</b>  | Activity-specific structuring                                | General education sequence (tell, apply, measure)      |
|            | <b>Justified Model Selection</b>                             | <b>“Naming Method, Not Justifying”</b>                 |
|            | Emphasis on “Children do what is shown”                      | Direct names like “Waldorf” or “active learning”       |
|            | Selecting approach according to learner level                | Lack of explanation on why and how it is applied       |
|            |  |  |
| <b>Q7</b>  | <b>Behavior-Based Observation and Analysis</b>               | <b>Observation-Based Direct Assessment</b>             |
|            | Tracking behavior instead of verbal expression               | Observation of application error                       |
|            | Task-based observation                                       | Observation + question-answer                          |
|            | Reaction to instruction                                      | Square drawing, object placement                       |
|            | <b>Process-Oriented Assessment</b>                           | <b>Result-Oriented Assessment</b>                      |
| <b>Q8</b>  | Asking questions at end of activity                          | Worksheets   |
|            | Daily review and comparison                                  | Homework control                                       |
|            | Diversifying questions                                       | Post-topic application                                 |
|            | <b>Misconcept Awareness</b>                                  | <b>Recognizing Misconceptions Without Examples</b>     |
|            | Mistakes stemming from prior learning                        | Wrong placement, confusion emphasis                    |
| <b>Q9</b>  | Testing with opposite concept                                | General inability to comprehend                        |
|            | Correction via concretization                                | <b>Instant Correction Reaction</b>                     |
|            | <b>Reflective Assessment Awareness</b>                       | Assessing by getting immediate answers to questions    |
|            | What did we learn? What did we do? questions                 | Repeating when wrong                                   |
|            | Discussion and review planning                               | Only in-class feedback                                 |
| <b>Q9</b>  | Emphasis on family cooperation                               |  |
|            |  |  |
|            |  |  |
|            |  |  |
|            |  |  |
| <b>Q8</b>  | <b>Program and Curriculum-Based Preparation</b>              | <b>Recourse to General Sources</b>                     |
|            | Preschool education program                                  | Activity books   |
|            | Education flow and lesson plans                              | Video content  |
|            | Maarif model   | University period books                                |
|            | <b>Scientific and Academic Resource Usage</b>                | <b>Daily Practical Content</b>                         |
| <b>Q9</b>  | Field articles, journals                                     | Social media posts                                     |
|            | Academic books and research                                  | General scanning, creative ideas                       |
|            | Literature review  | Teacher forums and personal observation                |
|            | <b>Applied Observation and Experience Transfer</b>           | <b>Personal Experience and Intuitive Sources</b>       |
|            | Observation, internship experience                           | Observation, intuition, creativity                     |
| <b>Q10</b> | Cluster cooperation  | Orientation based on personal experiences              |
|            | <b>Skill of Combining Multiple Sources</b>                   | <b>Planning Based on Singular Sources</b>              |
|            | Web + book + program + experience                            | Activity book + video                                  |
|            | Sample plan analysis   | Direct adaptation from auxiliary sources               |
|            |  |  |

The findings in Table 13 show that educators with high Math-PCK emphasized developmental appropriateness and generally linked abstract concepts to concrete strategies. For example, Y-5 stated: "Math activities are often abstract and difficult for preschoolers. Our goal is to simplify the concepts to their age." Similarly, Y-3 described addition as an "increasing action," while Y-7 emphasized the "one-to-one correspondence principle." Problem-solving and reasoning were also prominent only in this group; as in Y-2's question: "How many vegetables, how many fruits did you collect? If you eat one on the way, how many are left?" In contrast, educators with low Math-PCK set superficial goals. D-1 stated: "I would do addition and subtraction up to 10 in the activity," while others limited the goals to "teaching shapes" or "recognizing money." Although D-6 mentioned "helping children recognize and use money in daily life," the explanation lacked a deeper pedagogical rationale. Furthermore, participants with high Math-PCK associated mathematics with multiple developmental domains. Y-4 explained: "Listening, following instructions, distinguishing colors, collaborating in a group, fine motor skills..." Such multidimensional perspectives are not present in the lower group, which tends to emphasize only basic skills such as counting. In general, high Math-PCK educators articulate not only what they will teach but also why and how they will teach it, while low Math-PCK educators focus only on content delivery without a deeper pedagogical foundation.

Analysis of responses to the second question revealed significant differences between early childhood educators in the high and low Math-PCK groups. Educators in the high Math-PCK group offered multifaceted explanations addressing pedagogical, developmental, and experiential dimensions of the importance of mathematical concepts. They articulated not only what children should learn but also why and how it should be applied. For example, Y-5 stated: "Children are guided by the mathematical education they receive; concepts such as listing, grouping, paying, separating, increasing, and decreasing should be applicable in daily life and practiced correctly within this framework." In contrast, low Math-PCK educators emphasized the relevance of mathematics in daily life but provided limited pedagogical justification. For example, D-5 stated: "Mathematics is a part of our lives in our daily lives. Therefore, it is important to make it important even when slicing pasta or fruit." While accurate in content, such statements lack depth in explaining the learning process. High Math-PCK participants also emphasized the fundamental role of mathematical skills for future learning. Y-4 stated: "The acquisition of basic mathematical skills, such as counting, grouping, and matching, forms the foundation for a child's later mathematical education. These skills are prerequisites for advanced skills such as basic arithmetic, ordering, and comparison." However, lower-level Math-PCK educators only addressed general outcomes such as school readiness (e.g., D-1: "important within the elementary school readiness process"). Themes of meaningful understanding and internalization also differentiated the groups. Y-2 emphasized: "A group that internalizes the subject will achieve its goals faster." Higher-level Math-PCK educators, in Y-7, addressed conceptual accuracy, such as the principles of counting, stating: "To prevent and correct fundamental errors such as skipping counts or counting an object twice when performing counting operations. Thus, it represents meaningful counting and its principles for children." Finally, high-level Math-PCK educators emphasized holistic developmental aspects such as attention, thinking, language, and early literacy, as noted by Y-6: "Mathematical domain skills – the development of social feeling, Turkish domains, early literacy level are important in this period, in terms of preparation for primary school." Low-level Math-PCK responses occasionally referred to individual skills such as creativity or analytical thinking (e.g., D-4), but this lacked context or justification. In summary, high-level Math-PCK educators offered well-justified, multidimensional explanations that connected mathematical concepts to pedagogical, developmental, conceptual, and experiential aspects, while low-level Math-PCK educators' responses remained content-focused and superficially justified.

Analysis of responses to the third question revealed that high-level Math-PCK early childhood educators not only identified content excluded from children's learning but also provided pedagogical justifications for these omissions, demonstrating effective filtering between comprehensive subject knowledge and developmental pedagogy. For example, Y-4 stated: "As teachers, we know there are more detailed concepts behind the activity. That numbers start from zero, the properties of numbers, counting policies, naturally counting according to individual rules without repetition... But for children, this is not a rule, it is applied." This indicates that the educator is aware of abstract concepts but strategically chooses not to teach them directly, considering developmental appropriateness. Similarly, Y-2 emphasized implicit learning: "We may have implicit goals. We can give these superficially, without the children knowing, within the program or activity." In contrast, low-level Math-PCK educators described the exclusion of content with superficial or function-based reasoning. D-3 noted: "Even if they learn the indicative or diameter of geometric shapes, where to find them, so people at this age don't need to know that." Similarly, D-6 stated: "Other countries' currencies, images of the money those countries use... none of that preschool children need to know." These statements reflect knowledge of the content but lack integration with developmental or cognitive considerations. Lower Math-PCK participants, such as D-7, excluded content without justification: "I didn't include anything in the subject that children don't need to know." Higher Math-PCK educators consistently linked content exclusion to conceptual complexity, developmental appropriateness, learning theories, implicit teaching strategies, and pedagogical design. For example, Y-4 explained: "We can get

pre-operational and broad thinking at a concrete level from the children we're doing the activity with. This knowledge shapes our teaching style." Conversely, lower Math-PCK educators relied more on everyday experience, general assumptions, or superficial reasoning. In summary, educators with high Math-PCK scores demonstrate strategic, multi-dimensional decision-making skills in content selection, while the omissions of educators with low Math-PCK scores are based on more superficial and inadequate justifications.

Analysis of responses to the fourth question revealed that high Math-PCK early childhood educators attributed children's learning difficulties not only to knowledge or skill deficiencies but also to developmental, instructional, and environmental factors. For example, Y-4 stated: "Short attention spans... they may lose interest in repetitive activities such as counting and grouping. Number symbols may be listable, quantity concepts may be isolated. Some children may have difficulty understanding colors or instructions that cannot be expressed or supported." This statement links the difficulty to cognitive and language development. In contrast, low Math-PCK educators primarily focused on children's behavioral responses or difficulties in understanding concepts. D-1 stated: "Children could add up quantities, store quantities, but subtractions were a little difficult at times." Here, the underlying conceptual or pedagogical reasons for the difficulty were not addressed, making the response more superficial. High Math-PCK participants also considered the instructional context and material adequacy. Y-5 explained: "The fundamental problem is the lack of material and support from the Ministry of National Education in making abstract concepts concrete... the focus should be on materials, and deficiencies should be addressed." In contrast, lower-level Math-PCK educators rarely directly addressed material limitations, but some did mention children's inability to concretize concepts (e.g., D-6: "Having more coins makes children think they are more valuable... they can't grasp it."). Higher-level Math-PCK educators also emphasized instructional adaptations for individual differences, advocating for differentiation, personalization, and flexible strategies. Y-2 stated: "Enrichment or differentiation according to the group, activities can be tailored to groups. Adaptations can also be made for an internationally disabled student." Lower-level Math-PCK educators mentioned individual differences but primarily viewed them as challenges rather than offering solutions (e.g., D-7: "Some children acquire the learning easily, while for others I need to do extra work."). Finally, higher-level Math-PCK educators demonstrated awareness of long-term consequences, including failure and bias. Y-8 stated: "They mix things up... failure causes them to lose interest, and it forms a prejudice against mathematics." This highlights the capacity to anticipate the future impact of current difficulties, a perspective largely absent from low Math-PCK responses.

Analysis of responses to the fifth question revealed that early childhood educators with high Math-PCK scores explicitly expressed pedagogical awareness regarding children's thinking and conceptual understanding. Participants in this group recognized that children struggle with abstract concepts and emphasized the need for concrete materials to support learning. For example, Y-1 stated, "Abstract conceptual understanding is not understood in my timeframe, this limits me," reflecting a developmentally sensitive assessment. Educators with low Math-PCK scores also noted developmental differences, but these were generally described superficially as age-based adjustments. D-2 stated, "It responds immediately to the 5-year-old group... I simplify the subject for the 3-year-old group," reflecting content simplification rather than strategic pedagogy. High Math-PCK participants offered more in-depth explanations about pre-existing knowledge gaps and conceptual misunderstandings. For example, Y-5 highlighted the cognitive load involved in instruction by discussing how children confuse symbols and struggle with "+" and "-". In contrast, educators with low Math-PCK scores largely described the behavioral consequences of similar misconceptions (e.g., D-6: "They think a coin is more valuable because there are more of it"). Some educators with high Math-PCK scores also noted that children's curiosity and ideas enriched learning and that they adapted their plans accordingly. Y-4: "Curiosity and questioning increase a child's interest and facilitate the transition to new learning." At this point, low Math-PCK responses were limited, with D-5 simply stating "there are occasional brilliant ideas" without providing concrete examples. Finally, both groups acknowledged constraints such as attention span and limited time. However, while high Math-PCK educators discussed strategic planning to mitigate these challenges, low Math-PCK educators primarily presented descriptive observations.

When evaluating the responses to the sixth question, the High Math-PCK group presented practices based on play, interaction, developmental differences, and pedagogical justifications when explaining their teaching approaches. Y-4's explanation was multifaceted: "Play-based learning, moving from concrete to abstract, adapting activities according to individual differences, ensuring active participation." Such responses demonstrate that the early childhood educator knows not only the method but also why and how it is applied. The Low Math-PCK group, on the other hand, based their activity processes more on classic techniques such as "using concrete objects" and "learning by doing"; however, they were limited in justifying these or explaining them with a structured pedagogical approach. For example, D-1 stated: "I tried to teach using the demonstration technique. I sent homework assignments to reinforce the topic." While this explanation is technically correct, its pedagogical context is weak. Some participants in the High group stated that they used more than one approach situationally, and that they structured the activities flexibly and specifically for the child. Y-8 said: "We use many approaches depending on the situation." Such statements demonstrate that early childhood educators have internalized not

only theoretical but also practical knowledge of their approach. Even though different techniques were included in the lower group, some participants merely mentioned the name of an approach, such as "Waldorf," without establishing a connection between application and outcome (e.g., D-3). This indicates a superficial level of pedagogical content knowledge.

When evaluating the responses to the seventh question, the High Math-PCK group emphasizes both behavioral observation and process-oriented questioning in assessing whether children understand concepts. For example, Y-1 states, "I understand whether they understand the concept more from their behavior than from their verbal expression," indicating how understanding is monitored through behavioral indicators, going beyond classic verbal assessments. In contrast, the Low Math-PCK group generally uses more limited and outcome-oriented assessment methods based on observation and homework-checking. D-1 explains this situation as follows: "I sent homework assignments home... I observed what they learned by asking questions." This is an approach based on predicting understanding through repetition rather than measurement. Some participants in the High Math-PCK group defined misconceptions as systematic problems stemming from prior learning; Y-5, for example, said: "These misconceptions are usually children's incorrect prior learning... I tell them the opposite and ask them to find the correct one." Such strategies reflect the capacity for pedagogical intervention at the conceptual level. While early childhood educators in the lower Math-PCK group also addressed misconceptions, they generally expressed this as a simple observation of errors. D-8's statement is exemplary: "If they can't place the numbers correctly, I see that they don't understand." In the higher Math-PCK group, the use of reflective questions such as "What did we learn?" and "How did you feel?" at the end of the process is also noteworthy (Y-3, Y-2). This approach allows for both the recognition of children's thoughts and the collection of feedback regarding the activity process. In conclusion, while the assessment and misconception identification methods of early childhood educators in the higher Math-PCK group are more holistic, child-centered, and reasoned, participants in the lower group mostly use traditional, outcome-based approaches, offering limited examples in terms of pedagogical intervention.

When evaluating the responses to the last question, the High Math-PCK group indicated that they consulted both formal programs and scientific resources in the lesson preparation process and structured their material selection accordingly. For example, Y-6 stated: "I use articles, books, and journals related to mathematical skills." This shows that a bridge was built between an academic foundation and application. Early childhood educators in the Low Math-PCK group, on the other hand, stated that they mostly used activity books, videos, and general content; these resources were mostly based on adaptation and imitation-based planning. D-3 said: "By watching videos on the subject." However, this approach reflects a superficial, application-oriented orientation, far from establishing pedagogical cause-and-effect relationships. Some participants in the High group emphasized that they prepared lesson plans based on program integrity, educational flow, and activity objectives, with Y-2 stating: "University textbooks, our program books, our educational flows..." This shows that the activity process was handled in a planned and structured manner. Even in the lower group, while some participants mentioned university textbooks and preschool programs, it was observed that their references to these resources were limited and not supportive, but merely familiar content. This creates a deficiency in terms of content integrity.

## Conclusion and Discussion

The quantitative findings of the study showed that early childhood educators generally had a moderate level of pedagogical content knowledge (PCK) regarding mathematics. This aligns with recent national and international studies showing that while early childhood educators may possess adequate mathematical PCK, it often lacks depth (Aumann et al., 2024; Bilgen & Öztürk, 2023). Effectively presenting mathematical concepts requires not only conceptual knowledge but also the ability to convey this knowledge in a child-centered way (Clements & Sarama, 2020; Gonulates & Gilbert, 2023). However, many educators struggle to connect conceptual understanding with pedagogical decision-making (Gasteiger & Benz, 2018), which poses an obstacle to high-quality early childhood mathematics education. Consistent with these findings, most participants in this study gave content-focused responses with limited pedagogical justification and highlighted the need to improve PCK training implemented in teacher training programs. The study's second and third findings showed that early childhood educators' pedagogical content knowledge (PCK) regarding mathematics varied significantly according to specific demographic factors, particularly education level, type of institution, and age group of the children being taught, while age did not have a significant effect. These results are consistent with previous research suggesting that teacher quality can be influenced by contextual variables. Education level is considered one of the cornerstones of pedagogical knowledge, and higher education directly enhances conceptual and pedagogical skills (Kutluca, 2021; Lee, 2017). Educators in public institutions generally undergo more systematic in-service training and adhere more strictly to the formal curriculum (Argin & Dağlıoğlu, 2020; Orcan-Kaçan et al., 2023). Similarly, those working with older children strengthen the use of PCK by incorporating more systematic mathematical concepts (Clements et al., 2023; Gervasoni & Perry, 2015). In contrast, the lack of a significant effect of age highlights that experience alone is insufficient, emphasizing the need for systematic knowledge updating and pedagogical reflection (Evens et al., 2015).

The study revealed that early childhood educators with a high level of Math-PCK based their math activities on pedagogical principles such as developmental appropriateness, inter-conceptual relationships, and real-life contextualization. In contrast, educators in the low Math-PCK group tended to limit activity planning to content presentation and superficial application. This finding highlights that pedagogical content knowledge is not only related to the level of knowledge but also to how it is structured and applied in teaching contexts. Educators with a high Math-PCK level considered children's developmental characteristics, the relationships established between concepts, and relating activities to daily experiences—key indicators of quality early math education. As Clements and Sarama (2020) state, supporting early math learning with developmentally appropriate and meaningful experiences deepens children's conceptual understanding. Similarly, Pekince and Avcı (2016) emphasize that relating activities to children's life experiences increases learning retention. High-level Math-PCK educators, in line with Shulman's (1987) definition of pedagogical content knowledge, integrated pedagogical principles such as concretizing abstract concepts, one-to-one mapping, and grouping. They advocated not only what to teach but also why and how to teach, demonstrating strong pedagogical reasoning (Gasteiger & Benz, 2018; Kutluca & Mercan, 2022). Conversely, low-level Math-PCK educators primarily focused on content presentation, reducing learning to mere information transfer, which demonstrates a superficial level of pedagogical knowledge (Gülbağıci-Dede et al., 2023). This finding underscores the need to support educators in adopting a child-centered, developmentally informed, and holistic approach. Overall, the results demonstrate that high-quality early mathematics education requires not only content knowledge but also the pedagogically grounded structuring of that content.

The study's fifth finding demonstrates that early childhood educators' pedagogical content knowledge (Math-PCK) directly influences the quality of their teaching strategies. Educators with high Math-PCK levels justify their activities based on children's developmental characteristics, conceptual needs, and contextual realities, and design teaching strategies consciously and flexibly. By considering not only what to teach but also why and how to teach, they transform teaching from superficial activity presentations into meaningful processes that support children's mathematical thinking (Clements & Sarama, 2020). According to the learning trajectories approach, analyzing children's current knowledge to plan next steps is fundamental to effective teaching. Conversely, educators with low Math-PCK levels often rely on pre-prepared activities, resulting in teaching disconnected from children's cognitive development and contextual appropriateness (Akdeniz & Şimşek, 2022). This aligns with Argın and Dağlıoğlu's (2020) findings, which show that educators with low PAC often base their activities on materials or standard practices. Baroody et al. (2019) emphasize that early mathematics education is more than just content knowledge; it requires engaging children in play and exploration-based activities that develop problem-solving, classification, and patterning skills. Accordingly, educators with Higher Math-PCK structure their teaching strategies to support these dimensions of cognitive development (Vogt et al., 2020). Torbevens et al. (2021) also found that Higher Math-PCK encourages creative, flexible, and child-centered approaches in classroom practices. Overall, pedagogically grounded and theory-based teaching strategies enhance the quality of early mathematics education and deepen children's development of mathematical thinking.

The study's sixth finding demonstrates that early childhood educators' pedagogical content knowledge (Math-PCK) regarding mathematics significantly influences how they plan and structure mathematics activities. This finding highlights that PCK directly impacts the quality of teaching approaches in early childhood mathematics education. Educators with high Math-PCK view mathematics not only as cognitive content but also as learning opportunities tailored to children's developmental characteristics, interests, and contextual appropriateness, exemplifying Shulman's (2015) principle of transforming knowledge according to content, context, and learner. Alonso et al. (2019) emphasize that knowing what to teach is not enough; awareness of how to teach and guide children's meaning-making processes is crucial for effective learning. Similarly, the learning trajectories approach (Clements and Sarama, 2020) suggests that effective mathematics instruction requires activities that are developmentally appropriate, pedagogically meaningful, and contextually flexible. Therefore, high-level Math-PCK educators offer enriched mathematical experiences by considering children's prior knowledge, play-based learning opportunities, and everyday life contexts (Baroody et al., 2019; Gasteiger et al., 2020). In contrast, low-level Math-PCK educators focus primarily on content transfer (Aksu & Kul, 2017; Bilgen & Öztürk, 2023), limiting exploratory learning and reducing activities to superficial information transfer. Vogt et al. (2020) emphasize that such differences in teaching approach can significantly affect the development of children's mathematical thinking. Consequently, this finding highlight that the Math-PCK levels of early childhood educators determine not only their knowledge but also the pedagogical depth and structuring of that knowledge and underscores the importance of comprehensive and contextually grounded PAC development for high-quality early mathematics education.

The seventh finding of the study shows that early childhood educators with a high level of Math-PCK adopt an approach to mathematics education that prioritizes conceptual depth, is supported by reasoned explanations, and considers children's multiple developmental areas. These educators holistically consider cognitive, social-emotional, and language development when guiding children's mathematical thinking processes; thus, they treat

mathematical concepts not merely as information to be taught, but also as developmental learning opportunities (Baroody et al. 2019; Clements & Sarama, 2020; Vogt et al. 2020). In contrast, early childhood educators with a low level of Math-PCK often treat mathematics superficially, struggle to establish relationships between concepts, and present activities at an instrumental level, neglecting reasoning processes (Alonzo et al. 2019; Chan & Yung, 2018; Gasteiger & Benz, 2018). This situation suggests that the pedagogical content knowledge deficiencies of the group in question may limit children's ability to develop a deep mathematical understanding, and that the activity process may be largely hands-on but lacking a contextual basis (Bilgen & Öztürk, 2023; Ginsburg, 2016; Torbeyns et al., 2021).

The study's eighth finding demonstrates that early childhood mathematics education involves not only content delivery but also pedagogically complex, multidimensional decision-making. Educators with a high level of Math-PCK systematically implement higher-level pedagogical decisions such as setting latent learning objectives, ensuring developmental appropriateness, and selectively skipping concepts (Alonzo et al., 2019; Gasteiger et al., 2020; Shulman, 2015). These educators make informed choices about which mathematical content to emphasize or defer based on children's age, cognitive capacity, and readiness, reflecting strategic instructional planning (Clements & Sarama, 2020; Vogt et al., 2020). In contrast, educators with low Math-PCK levels fail to articulate such decisions or express them superficially, often relying on specific, content-focused routines (Avcı & Akman, 2023; Bilgen & Öztürk, 2023; Torbeyns et al., 2021). This suggests that educators with low PAB levels are less adept at making sensitive instructional decisions to support children's mathematical learning (Chan & Hume, 2019; Oppermann et al., 2016). Another finding reveals that the quality of assessment practices in early mathematics is determined by educators' Math-PCK levels. Educators with high Math-PCK levels, on the other hand, use multifaceted and dynamic assessment strategies to monitor children's mathematical development, analyze conceptual understanding, and identify misunderstandings (Alonzo et al., 2019; Clements & Sarama, 2020; Gasteiger & Benz, 2018). They conduct holistic assessments through informal observations, structured activities, learning outcome analysis, and individual interactions, viewing assessment not merely as an outcome but as a process that informs instruction (Baroody et al., 2019; Eynde et al., 2024; Torbeyns et al., 2021). Conversely, low-level Math-PCK educators tend to implement limited, superficial assessment practices, such as filling out observation forms or checking assignments, thus missing opportunities to gain rich insights into learning processes (Bilgen & Öztürk, 2023; Gonulates & Gilbert, 2023; Satan et al., 2024). These findings highlight that effective assessment is not merely a technical task but is directly linked to pedagogical knowledge and understanding and requires a high level of Math-PCK (Chan & Hume, 2019; Gess-Newsome et al., 2019).

The tenth finding of the study shows that early childhood educators' approaches and methods in mathematics teaching are directly linked to their levels of pedagogical content knowledge (Math-PCK). Educators with high Math-PCK not only name their teaching approaches but also explain how these approaches support children's developmental characteristics, the pedagogical justifications for their selection, and how they are integrated into classroom practices (Alonzo et al., 2019; Clements & Sarama, 2020; Shulman, 2015). This group structures activities in a flexible and child-centered way, basing them on play-based learning, inquiry-based activities, or learning trajectories (Alsina & Berciano, 2020; Gasteiger et al., 2020). On the other hand, educators with low Math-PCK levels often rely on traditional methods such as "lecturing," "demonstration," or "question-and-answer," and do not pedagogically address why or how questions (Bilgen & Öztürk, 2023; Lee, 2017; Orcan-Kaçan et al., 2023). This demonstrates that effective mathematics teaching requires not only knowledge of methods but also the ability to justify and adapt them based on pedagogical understanding (Gess-Newsome et al., 2019; Gonulates & Gilbert, 2023; Torbeyns et al., 2020). Furthermore, educators' resource choices for lesson planning are also related to their Math-PCK levels. High Math-PCK educators rely on curriculum guides, academic publications, learning pathways, and child observations instead of simply relying on readily available materials. This allows for both academically based and child-centered planning (Alonzo et al., 2019; Clements & Sarama, 2020; Gasteiger & Benz, 2018). This approach enables them to plan what, why, and how to teach, ensuring that learning is developmentally appropriate, conceptually consistent, and contextually meaningful (Fuson & Leinwand, 2023; Sarama et al., 2016). In contrast, educators with low Math-PCK often rely on superficial resources such as activity books or online videos, resulting in formulaic, pedagogically superficial, and insensitive lesson plans (Argın & Dağlıoğlu, 2020; Ban et al., 2024; Bilgen & Öztürk, 2023). Resource selection not only affects the quality of planning but also directly influences the effectiveness of teaching processes and the quality of children's learning experiences (Copur-Gençturk & Li, 2023; Pelkowski et al., 2019; Torbeyns et al., 2021). Therefore, Math-PCK emerges as a critical factor shaping both teaching methods and resource utilization. Based on the findings, some recommendations have been made regarding teacher training, practice, and research:

1. To improve the pedagogical content knowledge of early childhood educators regarding mathematics, workshops addressing mathematical concepts at a developmental level can be organized in in-service training programs.

2. Reflective activities can be included where early childhood educators can evaluate their own activity processes. This will help them recognize the connection between their activity practices and their pedagogical knowledge.
3. The fact that early childhood educators working in state institutions have a higher level of Math-PCK demonstrates the importance of in-house support and guidance. In this context, similar professional support mechanisms can be created for early childhood educators working in private institutions.
4. To reduce the dependence of early childhood educators on ready-made activities, material development studies can be carried out to support them in developing their own structures.
5. Specifically in early childhood mathematics education, guidance studies can be conducted to support the more effective use of curriculum books. In this way, early childhood educators can interpret existing resources on a more pedagogical basis.
6. Longitudinal studies can be conducted to observe how early childhood educators' pedagogical content knowledge regarding mathematics education is shaped from the early years of their professional lives and what factors influence it. This would allow for a clearer view of the impact of teacher training programs.
7. Quantitative and qualitative mixed-methods research can be conducted to understand the relationships between early childhood educators' attitudes towards mathematics, their self-efficacy perceptions, and their pedagogical content knowledge.
8. The contribution of mathematics applications integrated with other early childhood disciplines such as art, science, and play to the pedagogical content knowledge levels of early childhood educators can be investigated using experimental or quasi-experimental designs.

These recommendations can be guiding in ensuring that research findings contribute to the field and in supporting the quality of mathematics education in early childhood.

## DECLARATIONS

**Ethical Approval:** This study was conducted with permission granted by the XXX University Faculty of Educational Sciences Ethics Committee, decision number 2025/05, dated 22.05.2025.

**Conflict of Interest:** The authors declare that there are no financial or non-financial conflicts of interest that could affect this study.

**Funding:** No specific grant was received from any funding agency in the public, commercial, or not-for-profit sectors.

## References

Akdeniz, K., & Şimşek, N. (2022). Okul öncesi matematik öğretim bilgisi testi: Uyarlama ve uygulama çalışması. *Erken Çocukluk Çalışmaları Dergisi*, 6(1), 102-131.

Aksu, Z., & Kul, Ü. (2017). Turkish Adaptation of the Survey of Pedagogical Content Knowledge in Early Childhood Mathematics Education. *Online Submission*, 8(30), 1832-1848.

Alonzo, A. C., Berry, A., & Nilsson, P. (2019). Unpacking the complexity of science teachers' PCK in action: Enacted and personal PCK. In *Repositioning pedagogical content knowledge in teachers' knowledge for teaching science* (pp. 273-288). Singapore: Springer Nature Singapore.

Alsina, A., & Berciano, A. (2018). Developing informal mathematics in early childhood education. *Early Child Development and Care*, 190(13), 2013–2031.

Argın, Y., & Dağlıoğlu, H. E. (2020). An investigation into mathematics-related pedagogical content knowledge of preschool educators based on institution type. *İlköğretim Online*, 19(4), 1948-1962.

Aumann, L., Puca, R. M., & Gasteiger, H. (2025). Effects of Early Childhood Teacher Knowledge and Feedback on Children's Early Mathematical Development in German Kindergartens. *Early Childhood Education Journal*, 1-11.

Bakken, L., Brown, N., & Downing, B. (2017). Early childhood education: The long-term benefits. *Journal of research in Childhood Education*, 31(2), 255-269.

Ban, J., Msall, C., Douglas, A. A., Rittle-Johnson, B., & Laski, E. V. (2024). Knowing what they know: Preschool teachers' knowledge of math skills and its relation to instruction. *Journal of Experimental Child Psychology*, 246, 105996.

Baroody, A. J., Clements, D. H., & Sarama, J. (2019). Teaching and learning mathematics in early childhood programs. *Handbook of early childhood care and education*, 1, 329-353.

Bilgen, Z., & Öztürk, Y. A. (2023). Investigation of pedagogical content knowledge of in-service and pre-service pre-school teachers in pre-school mathematics, *International Journal of Progressive Education*, 19(5), 126-152.

Björklund, C., van den Heuvel-Panhuizen, M., & Kullberg, A. (2020). Research on early childhood mathematics teaching and learning. *ZDM*, 52(4), 607-619.

Büyüköztürk, Ş. (2022). Sosyal bilimler için veri analizi el kitabı. *Pegem Atıf İndeksi*, 001-214.

Chan, K. K. H., & Hume, A. (2019). Towards a consensus model: Literature review of how science teachers' pedagogical content knowledge is investigated in empirical studies. *Repositioning pedagogical content knowledge in teachers' knowledge for teaching science*, 3-76.

Chan, K. K. H., & Yung, B. H. W. (2018). Developing pedagogical content knowledge for teaching a new topic: More than teaching experience and subject matter knowledge. *Research in Science Education*, 48(2), 233-265.

Clements, D. H., & Sarama, J. (2014). Developing young children's mathematical thinking and understanding. In *The Routledge international handbook of young children's thinking and understanding* (pp. 331-344). Routledge.

Clements, D., & Sarama, J. (2015). Discussion from a mathematics education perspective. *Mathematical Thinking and Learning*, 17(2-3), 244-252.

Clements, D. H., & Sarama, J. (2020). *Learning and teaching early math: The learning trajectories approach*. Routledge.

Clements, D. H., Lizcano, R., & Sarama, J. (2023). Research and pedagogies for early math. *Education Sciences*, 13(8), 839.

Copur-Gençturk, Y., & Li, J. (2023). Teaching matters: A longitudinal study of mathematics teachers' knowledge growth. *Teaching and teacher education*, 121, 103949.

Creswell, J. W. (2021). *A concise introduction to mixed methods research*. SAGE publications.

Creswell, J. W., & Plano-Clark, V. L. (2017). *Designing and conducting mixed methods research*. Sage publications.

Creswell, J. W., & Poth, C. N. (2016). *Qualitative inquiry and research design: Choosing among five approaches*. Sage publications.

Dağlı, H., Dağlıoğlu, H. E., & Atalmış, E. H. (2019). Development of a preschool teachers' pedagogical content knowledge scale regarding mathematics. *International Journal of Assessment Tools in Education*, 6(4), 617-635.

Demirbaş, M. (2019). *Okul öncesi öğretmen adayları ve okul öncesi öğretmenlerinin matematiksel inanç düzeyleri ile matematiksel pedagojik yeterlik düzeyleri arasındaki ilişkinin incelenmesi* (Yayınlanmamış Yüksek Lisans Tezi). Necmettin Erbakan Üniversitesi, Eğitim Bilimleri Enstitüsü, Konya.

Elo, S., & Kyngäs, H. (2008). The qualitative content analysis process. *Journal of advanced nursing*, 62(1), 107-115.

Evens, M., Elen, J., & Depaepe, F. (2015). Developing pedagogical content knowledge: Lessons learned from intervention studies. *Education Research International*, 2015(1), 790417.

Op 't Eynde, E., Lara Mascareño, M., Depaepe, F., Verschaffel, L., & Torbeyns, J. (2024). Teacher-child interactions during shared book reading in the domain of early mathematics. *Mathematics Education Research Journal*, 1-28.

Fraenkel, J. R., Wallen, N. E., & Hyun, H. H. (2012). *How to design and evaluate research in education* (8th ed.). New York, NY: McGraw-Hill.

Frary, R. B. (1989). Partial-credit scoring methods for multiple-choice tests. *Applied Measurement in Education*, 2(1), 79-96.

Fuson, K. C., & Leinwand, S. (2023). Building equitable math talk classrooms. *Mathematics Teacher: Learning and Teaching PK-12*, 116(3), 164-173.

Gasteiger, H., Bruns, J., Benz, C., Brunner, E., & Sprenger, P. (2020). Mathematical pedagogical content knowledge of early childhood teachers: A standardized situation-related measurement approach. *ZDM*, 52(2), 193-205.

Gasteiger, H., & Benz, C. (2018). Enhancing and analyzing kindergarten teachers' professional knowledge for early mathematics education. *The Journal of Mathematical Behavior*, 51, 109-117.

Gejard, G., & Melander, H. (2020). Mathematizing in preschool: Children's participation in geometrical discourse. In *Innovative Approaches in Early Childhood Mathematics* (pp. 33-49). Routledge.

Gervasoni, A., & Perry, B. (2015). Children's mathematical knowledge prior to starting school and implications for transition. In *Mathematics and transition to school: International perspectives* (pp. 47-64). Singapore: Springer Singapore.

Gess-Newsome, J., Taylor, J. A., Carlson, J., Gardner, A. L., Wilson, C. D., & Stuhsatz, M. A. (2019). Teacher pedagogical content knowledge, practice, and student achievement. *International Journal of Science Education*, 41(7), 944-963.

Ginsburg, H. P. (2016). Helping early childhood educators to understand and assess young children's mathematical minds. *ZDM*, 48(7), 941-946.

Glaser, B., & Strauss, A. (2017). *Discovery of grounded theory: Strategies for qualitative research*. Routledge.

Gonulates, F., & Gilbert, J. (2023). Facilitating Effective Mathematical Teaching Practices in Preschool. *Journal of Global Education and Research*, 7(3), 265-279.

Gökçen, M., & Kutluca, A. Y. (2022). Okul öncesi öğretmenlerinin pedagojik inançlarının pedagojik yeterlikleri üzerindeki etkisinin incelenmesi. *Mersin Üniversitesi Eğitim Fakültesi Dergisi*, 18(3), 214-240.

Gülbağcı Dede, H., Yılmaz, Z., Akkoç, H., & Tall, D. (2023). Enhancing pre-service mathematics teachers' understanding of function ideas. *International journal of mathematical education in science and technology*, 54(10), 2122-2146.

Howard, S. J., Neilsen-Hewett, C., de Rosnay, M., Melhuish, E. C., & Buckley-Walker, K. (2022). Validity, reliability and viability of pre-school educators' use of early years toolbox early numeracy. *Australasian Journal of Early Childhood*, 47(2), 92-106.

Ivankova, N. V., Creswell, J. W., & Stick, S. L. (2006). Using mixed-methods sequential explanatory design: From theory to practice. *Field methods*, 18(1), 3-20.

Johnson, R. B., & Onwuegbuzie, A. J. (2004). Mixed methods research: A research paradigm whose time has come. *Educational researcher*, 33(7), 14-26.

Karakaş, T. B. & Kutluca, A. Y. (2025). Okul öncesi öğretmenlerinin bütünlendirilmiş fen ve matematik etkinliklerine ilişkin pedagojik yeterliliklerinin pedagojik inançları açısından incelenmesi. *Fen Eğitimi Araştırmalarına Güncel Bakış* içinde (ss. 73-94). Akademisyen Kitabevi

Kim, S., Jung, D., Kang, J., Kim, D., Kim, M., Park, J., ... & Lee, M. (2024). The Effects of Pre-service Early Childhood Teachers' Pedagogical Content Knowledge in Mathematics and Constructivist Educational Belief on Mathematics Teaching Efficacy. *International Journal of Advanced Culture Technology*, 441-448.

Kind, V., & Chan, K. K. (2019). Resolving the amalgam: connecting pedagogical content knowledge, content knowledge and pedagogical knowledge. *International Journal of Science Education*, 41(7), 964-978.

Konca, A. S., & Özçakır, B. (2021). Okul öncesi öğretmenlerinin ve öğretmen adaylarının matematiksel gelişim inancı ve erken çocukluk matematik pedagojik alan bilgisi. *Cumhuriyet Uluslararası Eğitim Dergisi*, 10(4), 1648-1671.

Kutluca, A. Y. (2021). An investigation of elementary teachers' pedagogical content knowledge for socioscientific argumentation: The effect of a learning and teaching experience. *Science Education*, 105(4), 743-775.

Kutluca, A. Y., & Mercan, N. (2022). Exploring the effects of preschool teachers' epistemological beliefs on content-based pedagogical conceptualizations and PCK integrations towards science teaching. *European Journal of Science and Mathematics Education*, 10(2), 170-193.

Lee, J. E. (2017). Preschool teachers' pedagogical content knowledge in mathematics. *International Journal of Early Childhood*, 49(2), 229-243.

Lincoln, Y. ve Guba, E. (1985). *Naturalistic inquiry: Establishing Trustworthiness*. Beverly Hills.

Loughran, J., Mulhall, P., & Berry, A. (2004). In search of pedagogical content knowledge in science: Developing ways of articulating and documenting professional practice. *Journal of Research in Science Teaching*, 41(4), 370-391.

Lundqvist, J., Franzén, K., & Munter, A. C. (2023). Early childhood mathematics: a case study. *Early Years*, 43(4-5), 763-777.

Mayring, P. (2021). *Qualitative content analysis: A step-by-step guide*. Sage.

Miles, M. B. & Huberman, A. M. (1994). Qualitative data analysis: An expanded sourcebook. *Thousand Oaks*.

National Council of Teachers of Mathematics (NCTM). (2000). *Principles and standards for school mathematics*. Reston: The National Council of Teachers of Mathematics Inc.

Nilsson, P. (2014). When teaching makes a difference: Developing science teachers' pedagogical content knowledge through learning study. *International journal of science education*, 36(11), 1794-1814.

Oppermann, E., Anders, Y., & Hachfeld, A. (2016). The influence of preschool teachers' content knowledge and mathematical ability beliefs on their sensitivity to mathematics in children's play. *Teaching and Teacher Education*, 58, 174-184.

Orcan-Kacan, M., Dedeoglu-Aktug, N., & Alpaslan, M. M. (2023). Teachers' mathematics pedagogical content knowledge and quality of early mathematics instruction in Turkey. *South African Journal of Education*, 43(4), 1-19.

Orçan-Kaçan, M. & Karayol, S. (2017). Okul öncesi öğretmenlerin matematik eğitimi için ayırdıkları süre ve matematik eğitimi ile ilişkin görüşleri. *Uluslararası Eğitim Bilimleri Dergisi*, (12), 172-186.

Özdemir, B. (2020). Okul öncesi öğretmenlerinin erken matematik eğitimi ile ilişkin pedagojik alan bilgileri ile özyeterlilikleri arasındaki ilişkinin incelenmesi (Yayınlanmamış Yüksek Lisans Tezi). İstanbul Sabahattin Zaim Üniversitesi Lisansüstü Eğitim Enstitüsü, İstanbul.

Palinkas, L. A., Horwitz, S. M., Green, C. A., Wisdom, J. P., Duan, N., & Hoagwood, K. (2015). Purposeful sampling for qualitative data collection and analysis in mixed method implementation research. *Administration and policy in mental health and mental health services research*, 42(5), 533-544.

Pallant, J. (2020). *SPSS survival manual: A step by step guide to data analysis using IBM SPSS*. Routledge.

Papandreou, M., & Tsioli, M. (2022). Noticing and understanding children's everyday mathematics during play in early childhood classrooms. *International Journal of Early Years Education*, 30(4), 730-747.

Patton, M. Q. (2014). *Qualitative research & evaluation methods: Integrating theory and practice*. Sage publications.

Pekince, P., & Avcı, N. (2016). Okul öncesi öğretmenlerinin erken çocukluk matematiği ile ilgili uygulamaları: Etkinlik planlarına nitel bir bakış. *Kastamonu Eğitim Dergisi*, 24(5), 2391-2408.

Pelkowski, T., Herman, E., Trahan, K., Winters, D. M., Tananis, C., Swartz, M. I., ... & Rodgick, C. (2019). Fostering a "head start" in math: observing teachers in early childhood mathematics engagement. *Journal of Early Childhood Teacher Education*, 40(2), 96-119.

Sarama, J., Clements, D. H., Wolfe, C. B., & Spitler, M. E. (2016). Professional development in early mathematics: Effects of an intervention based on learning trajectories on teachers' practices. *Nordic Studies in Mathematics Education*, 21(4), 29-55.

Satan, N., Aksakal, K., Bağdat, A., & Altay, M. K. (2024). Preschool Teachers' Pedagogical Content Knowledge Regarding the Concept of Numbers. *Cukurova University Faculty of Education Journal*, 53(2), 652-680.

Sayan, G. (2023). *Okul öncesi öğretmenlerinin matematik pedagojik alan bilgileri ile çocukların çalışma belleği arasındaki ilişkinin incelenmesi* (Yayınlanmamış Yüksek Lisans Tezi). Hacettepe Üniversitesi Eğitim Bilimleri Enstitüsü Ankara.

Selvi, A. F. (2019). Qualitative content analysis. In *The Routledge handbook of research methods in applied linguistics* (pp. 440-452). Routledge.

Shulman, L. (1987). Knowledge and teaching: Foundations of the new reform. *Harvard educational review*, 57(1), 1-23.

Shulman, L. S. (2015). PCK: Its genesis and exodus. In *Re-examining pedagogical content knowledge in science education* (pp. 3-13). Routledge.

Tian, F., & Huang, J. (2019). Early childhood teachers' pedagogical content knowledge in mathematics: A research report from China. *Universal Journal of Educational Research*, 7(11), 2258-2261. <http://dx.doi.org/10.13189/ujer.2019.071102>

Tisdell, E. J., Merriam, S. B., & Stuckey-Peyrot, H. L. (2025). *Qualitative research: A guide to design and implementation*. John Wiley & Sons.

Torbeyns, J., Op't Eynde, E., Depaepe, F., & Verschaffel, L. (2024). Preschool teachers' mathematical questions during shared picture book reading. *ZDM-Mathematics Education*, 56(5), 907-921.

Vogt, F., Hauser, B., Stebler, R., Rechsteiner, K., & Urech, C. (2020). Learning through play-pedagogy and learning outcomes in early childhood mathematics. In *Innovative approaches in early childhood mathematics* (pp. 127-141). Routledge.

Whittaker, J. V., Kinzie, M. B., Vitiello, V., DeCoster, J., Mulcahy, C., & Barton, E. A. (2020). Impacts of an early childhood mathematics and science intervention on teaching practices and child outcomes. *Journal of Research on Educational Effectiveness*, 13(2), 177-212.

Willoughby, M., Hudson, K., Hong, Y., & Wylie, A. (2021). Improvements in motor competence skills are associated with improvements in executive function and math problem-solving skills in early childhood. *Developmental Psychology*, 57(9), 1463.

Yazlık, D. Ö., & Öngören, S. (2018). Okul öncesi öğretmenlerinin matematik etkinliklerine ilişkin görüşlerinin ve sınıf içi uygulamalarının incelenmesi. *Ahi Evran Üniversitesi Kırşehir Eğitim Fakültesi Dergisi*, 19(2), 1264-1283.

Yıldırım, A., & Simsek, H. (2021). *Sosyal bilimlerde nitel araştırma yöntemleri*. Seçkin Yayıncılık.

Youmans, A., Coombs, A., & Colgan, L. (2018). Early childhood educators' and teachers' early mathematics education knowledge, beliefs, and pedagogy. *Canadian Journal of Education/Revue canadienne de l'éducation*, 41(4), 1079-1104.

Zhang, W., Li, L., & Disney, L. (2024). Chinese early childhood teachers' perspectives on mathematics education in play-based contexts. *Early Years*, 44(3-4), 765-780.